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COSC 3020

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Homework 2

**1 Theory vs. Practice (9 points)**

1. List 3 reasons why asymptotic analysis may be misleading with respect to actual performance in practice. (3 points)

Big-O describes the growth of an algorithm at infinite input sizes, i.e. an algorithm that takes exactly 1 million steps per n is equivalent to one that takes 1 step per n. That is to say, 1 000 000n = O(n) and O(n) < O(n2). So if an algorithm A takes n2 steps per n, and B takes 1 million per, their asymptotic analysis says that O(A) < O(B) even though for all inputs less than 1,000,000 algorithm B is faster.

Though memory complexity is considered in asymptotic analysis, the speed of memory operations is not differentiated so long as they take some constant time. The same algorithm could have implementation A which uses a contiguous list in memory, while implementation B could be implemented as a linked-list split which can be split across memory. They both can have the same time complexity and memory complexity, though the performance of implementation A is likely able to be a lot faster.

Depending on the programming language, implementations of an algorithm can noticeably differ in performance between their recursive and iterative implementations.

1. Suppose finding a particular element in a binary search tree with 1,000 elements takes 5 seconds. Given what you know about the asymptotic complexity of search in a binary search tree, how long would you guess finding the same element in a search tree with 10,000 elements takes? Explain your reasoning. (3 points)
2. You measure the time with 10,000 elements and it takes 100 seconds! List 3 reasons why this could be the case, given that reasoning with the asymptotic complexity suggests a different time. (3 points)

**2 Graph Properties (9 points)**

1. Prove that if two graphs A and B do not have the same number of nodes, they cannot be isomorphic. (3 points)

Proof by contradiction

Definition of isomorphism:   
G1 = (V1, E1) is isomorphic to G2 = (V2, E2) if there is a one-to-one and onto function (bijection) f : V1 → V2 such that (u, v) ∈ E1 iff (f(u), f(v)) ∈ E2.

Graphs A and B do not have the same number of nodes:   
 A = (V, E), |V| = n  
 B = (X, Y), |X| = k   
 where k != n

If A and B are isomorphic, and n < k:  
 f() is a one-to-one function and all v ∈ V have been mapped to different x ∈ X  
 Therefore f() cannot be an ‘onto’ function because exists an x ∈ X that has not mapped to and there are no more v ∈ V to map without having repeats.

If A and B are isomorphic, and n > k:  
 f() is an onto function and all x ∈ X have been mapped onto by different v ∈ V  
 Therefore f() cannot be a ‘one-to-one’ function because exists an v ∈ V that has not mapped  
 to any x ∈ X and doing so requires repeat mappings.

Contradiction:  
f() cannot be both one-to-one and onto, a bijection f() does not exist between any graph A and B that do not have the same number of nodes, meaning that they cannot be isomorphic.

1. Prove that if two graphs A and B have the same number of nodes and are completely connected, they must be isomorphic. (3 points)
2. Prove that if two graphs A and B are isomorphic they do not have to be completely connected. (3 points) You need to give complete, formal proofs – state all your assumptions and make sure that you’ve explained every step of your reasoning. Hint: A good way to start is by writing down the definitions for everything related to what you want to prove.